Chapter 10 - Circles Excercise Ex. 10.1

Solution 1

(i)   The centre of a circle lies in interior of the circle. (exterior/interior)  
(ii)  A point, whose distance from the centre of a circle is greater than its radius lies in exterior of

      the circle. (exterior/interior)  
(iii) The longest chord of a circle is a diameter of the circle.  
(iv) An arc is a semicircle when its ends are the ends of a diameter.  
(v) Segment of a circle is the region between an arc and chord of the circle.  
(vi) A circle divides the plane, on which it lies, in three parts.

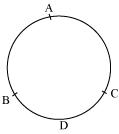
Solution 2

(i)  True, all the points on circle are at equal distance from the centre of circle, and this equal distance

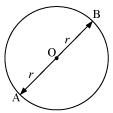
 it called as radius of circle.

(ii) False, on a circle there are infinite points. So, we can draw infinite number of chords of given length. Hence, a circle has infinite number of equal chords.

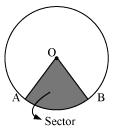
(iii) False, consider three arcs of same length as AB, BC and CA. Now we may observe that for minor arc BDC. CAB is major arc. So AB, BC and CA are minor arcs of circle.



(iv) True, let AB be a chord which is twice as long as its radius. In this situation our chord will be passing through centre of circle. So it will be the diameter of circle.



(v) False, sector is the region between an arc and two radii joining the centre to the end points of the arc as in the given figure OAB is the sector of circle.



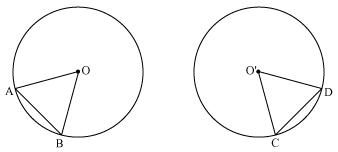
(vi) True, A circle is a two dimensional figure and it can also be referred as plane figure.

Chapter 10 - Circles Excercise Ex. 10.2

Solution 1

A circle is a collection of points which are equidistant from a fix point. This fix point is called as the centre of circle and this equal distance is called as radius of circle. And thus shape of a circle depends on the radius of the circle.  
            
So, if we try to superimpose two circles of equal radius, one each other both circles will cover each other.  
So, two circles are congruent if they have equal radius.

Now consider two congruent circles having centre O and O' and two chords AB and CD of equal lengths

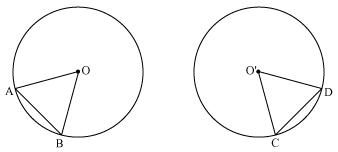


Now in http://images.topperlearning.com/topper/bookquestions/1134_ec7fd5b2b4109644a398eba4d25ebf00.pngAOB and http://images.topperlearning.com/topper/bookquestions/1134_ec7fd5b2b4109644a398eba4d25ebf00.pngCO'D  
AB = CD            (chords of same length)  
OA = O'C            (radii of congruent circles)  
OB = O'D            (radii of congruent circles)  
http://images.topperlearning.com/topper/bookquestions/1134_im19.JPG http://images.topperlearning.com/topper/bookquestions/1134_ec7fd5b2b4109644a398eba4d25ebf00.pngAOB http://images.topperlearning.com/topper/bookquestions/1134_3048037af7aaa664a9d374c13f47ecfe.png http://images.topperlearning.com/topper/bookquestions/1134_ec7fd5b2b4109644a398eba4d25ebf00.pngCO'D        (SSS congruence rule)

http://images.topperlearning.com/topper/bookquestions/1134_13fb2c7abb7fb90f3d38dc838d646ea3.png http://images.topperlearning.com/topper/bookquestions/1134_4934c411dbc1a55c0b7875b3e0fc862a.pngAOB = http://images.topperlearning.com/topper/bookquestions/1134_4934c411dbc1a55c0b7875b3e0fc862a.pngCO'D            (by CPCT)  
Hence equal chords of congruent circles subtend equal angles at their centres.

Solution 2

Let us consider two congruent circles (circles of same radius) with centres as O and O'.

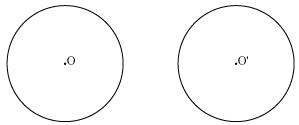


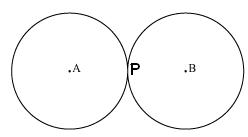
In http://images.topperlearning.com/topper/bookquestions/1136_ec7fd5b2b4109644a398eba4d25ebf00.pngAOB and http://images.topperlearning.com/topper/bookquestions/1136_ec7fd5b2b4109644a398eba4d25ebf00.pngCO'D  
http://images.topperlearning.com/topper/bookquestions/1136_4934c411dbc1a55c0b7875b3e0fc862a.pngAOB = http://images.topperlearning.com/topper/bookquestions/1136_4934c411dbc1a55c0b7875b3e0fc862a.pngCO'D        (given)  
OA = O'C            (radii of congruent circles)  
OB = O'D            (radii of congruent circles)  
 http://images.topperlearning.com/topper/bookquestions/1136_im19.JPGhttp://images.topperlearning.com/topper/bookquestions/1136_ec7fd5b2b4109644a398eba4d25ebf00.pngAOB http://images.topperlearning.com/topper/bookquestions/1136_6cf26576c060f5f190df510e5535ff3c.pngCO'D        (SSS congruence rule)  
http://images.topperlearning.com/topper/bookquestions/1136_13fb2c7abb7fb90f3d38dc838d646ea3.png AB = CD            (by CPCT)  
Hence, if chords of congruent circles subtend equal angles at their centres then chords are equal.

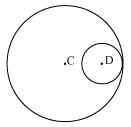
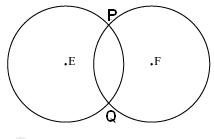
Chapter 10 - Circles Excercise Ex. 10.3

Solution 1

Consider the following pair of circles.  
(i) circles don't intersect each other at any point, so circles are not having any point in common.



(ii) Circles touch each other only at one point P so there is only 1 point in common.  
                                   
                     

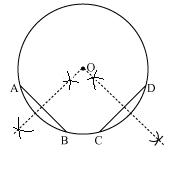
(iii) Circles touch each other at 1 point X only. So the circles have 1 point in common.       
  
                                                        
   
  
(iv) These circles intersect each other at two points P and Q. So the circles have two points in common. We may observe that there can be maximum 2 points in common.       
  
                         

We can have a situation in which two congruent circles are superimposed on each other, this situation can be referred as if we are drawing circle two times.

Solution 2

Following are the steps of construction:    
  
Step1. Take the given circle centered at point O.  
Step2. Take any two different chords AB and CD of this circle and draw perpendicular bisectors of these

          chords.  
Step3. Let these perpendicular bisectors meet at point O. Now, O is the centre of given circle.

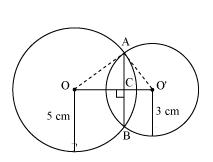


Solution 3

Consider two circles centered at point O and O' intersect each other at point A and B respectively.  
Join AB. AB is the chord for circle centered at O, so perpendicular bisector of AB will pass through O.  
Again AB is also chord of circle centered at O', so, perpendicular bisector of AB will also pass through O'.  
Clearly centres of these circles lie on the perpendicular bisector of common chord.

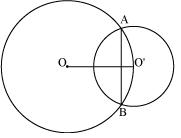
Chapter 10 - Circles Excercise Ex. 10.4

Solution 1



Let radius of circle centered at O and O' be 5 cm and 3 cm respectively.  
    OA = OB = 5 cm  
    O'A = O'B = 3 cm  
    OO' will be the perpendicular bisector of chord AB.  
    http://images.topperlearning.com/topper/bookquestions/1143_im19.JPG AC = CB

    Given that OO' = 4 cm  
    Let OC be x. so, O'C will be 4 - x  
    In http://images.topperlearning.com/topper/bookquestions/1143_ec7fd5b2b4109644a398eba4d25ebf00.pngOAC  
    OA2 = AC2 + OC2    
    http://images.topperlearning.com/topper/bookquestions/1143_13fb2c7abb7fb90f3d38dc838d646ea3.png 52 = AC2 + x2  
    http://images.topperlearning.com/topper/bookquestions/1143_13fb2c7abb7fb90f3d38dc838d646ea3.png 25 - x2 = AC2            ... (1)  
    In http://images.topperlearning.com/topper/bookquestions/1143_ec7fd5b2b4109644a398eba4d25ebf00.pngO'AC  
    O'A2 = AC2 + O'C2  
    http://images.topperlearning.com/topper/bookquestions/1143_13fb2c7abb7fb90f3d38dc838d646ea3.png 32 = AC2 + (4 - x)2  
    http://images.topperlearning.com/topper/bookquestions/1143_13fb2c7abb7fb90f3d38dc838d646ea3.png 9 = AC2 + 16 + x2 - 8x  
    http://images.topperlearning.com/topper/bookquestions/1143_13fb2c7abb7fb90f3d38dc838d646ea3.png AC2 = - x2 - 7 + 8x        ... (2)    
From equations (1) and (2), we have  
    25 - x2 = - x2 - 7 + 8x  
           8x = 32  
             x = 4  
So, the common chord will pass through the centre of smaller circle i.e. O'. and hence it will be diameter of smaller circle.

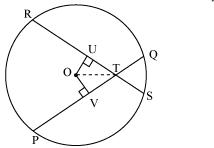


Now, AC2 = 25 - x2 = 25 - 42 = 25 - 16 = 9  
http://images.topperlearning.com/topper/bookquestions/1143_im19.JPG AC = 3 m

The length of the common chord AB = 2 AC = (2 http://images.topperlearning.com/topper/bookquestions/1143_b9ccb383b0c73c26cfb3ff194a86e235.png 3) m = 6 m

Solution 2

Let PQ and RS are two equal chords of a given circle and there are intersecting each other at point T.



    Draw perpendiculars OV and OU on these chords.  
    In http://images.topperlearning.com/topper/bookquestions/1144_ec7fd5b2b4109644a398eba4d25ebf00.pngOVT and http://images.topperlearning.com/topper/bookquestions/1144_ec7fd5b2b4109644a398eba4d25ebf00.pngOUT  
    OV = OU                                (Equal chords of a circle are equidistant from the centre)  
   http://images.topperlearning.com/topper/bookquestions/1144_4934c411dbc1a55c0b7875b3e0fc862a.pngOVT = http://images.topperlearning.com/topper/bookquestions/1144_4934c411dbc1a55c0b7875b3e0fc862a.pngOUT                     (Each 90o)  
   OT = OT                                (common)

http://images.topperlearning.com/topper/bookquestions/1144_im19.JPGhttp://images.topperlearning.com/topper/bookquestions/1144_ec7fd5b2b4109644a398eba4d25ebf00.pngOVT http://images.topperlearning.com/topper/bookquestions/1144_3048037af7aaa664a9d374c13f47ecfe.png http://images.topperlearning.com/topper/bookquestions/1144_ec7fd5b2b4109644a398eba4d25ebf00.pngOUT              (RHS congruence rule)

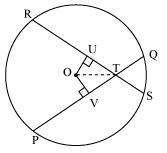
http://images.topperlearning.com/topper/bookquestions/1144_im19.JPG VT = UT                             (by CPCT)        ... (1)

    It is given that   
    PQ = RS                                           ... ... ... ... (2)

http://images.topperlearning.com/topper/bookquestions/1144_13fb2c7abb7fb90f3d38dc838d646ea3.pnghttp://images.topperlearning.com/topper/bookquestions/1144_cb3cfe781f270733f4acbe2739f2e845.png

http://images.topperlearning.com/topper/bookquestions/1144_13fb2c7abb7fb90f3d38dc838d646ea3.png PV = RU                                    ... ... ...  ... (3)  
    On adding equations (1) and (3), we have  
    PV + VT = RU + UT  
http://images.topperlearning.com/topper/bookquestions/1144_13fb2c7abb7fb90f3d38dc838d646ea3.png PT = RT                                    ... ... ...   ... (4)  
    On subtracting equation (4) from equation (2), we have  
    PQ - PT = RS - RT  
http://images.topperlearning.com/topper/bookquestions/1144_13fb2c7abb7fb90f3d38dc838d646ea3.png QT = ST                                      ... ... ... ... (5)  
    Equations (4) and (5) shows that the corresponding segments of  
    chords PQ and RS are congruent to each other.

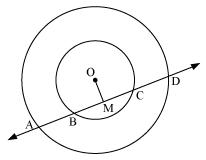
Solution 3



     Let PQ and RS are two equal chords of a given circle and there are intersecting each other at point T.  
     Draw perpendiculars OV and OU on these chords.  
     In http://images.topperlearning.com/topper/bookquestions/1146_ec7fd5b2b4109644a398eba4d25ebf00.pngOVT and http://images.topperlearning.com/topper/bookquestions/1146_ec7fd5b2b4109644a398eba4d25ebf00.pngOUT  
     OV = OU                           (Equal chords of a circle are equidistant from the centre)  
    http://images.topperlearning.com/topper/bookquestions/1146_4934c411dbc1a55c0b7875b3e0fc862a.pngOVT = http://images.topperlearning.com/topper/bookquestions/1146_4934c411dbc1a55c0b7875b3e0fc862a.pngOUT                 (Each 90o)  
    OT = OT                            (common)  
http://images.topperlearning.com/topper/bookquestions/1146_im19.JPGhttp://images.topperlearning.com/topper/bookquestions/1146_ec7fd5b2b4109644a398eba4d25ebf00.pngOVT http://images.topperlearning.com/topper/bookquestions/1146_1761c87e2ffbc82d2d19eafa2b97f1ee.pngOUT            (RHS congruence rule)  
http://images.topperlearning.com/topper/bookquestions/1146_im19.JPGhttp://images.topperlearning.com/topper/bookquestions/1146_4934c411dbc1a55c0b7875b3e0fc862a.pngOTV = http://images.topperlearning.com/topper/bookquestions/1146_4934c411dbc1a55c0b7875b3e0fc862a.pngOTU                 (by CPCT)

    Hence, the line joining the point of intersection to the centre makes equal angles with the chords.

Solution 4

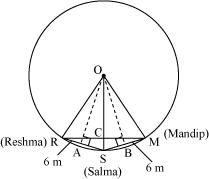
Let us draw a perpendicular OM on line AD.  
  
                     

    Here, BC is chord of smaller circle and AD is chord of bigger circle.  
    We know that the perpendicular drawn from centre of circle bisects the chord.  
http://images.topperlearning.com/topper/bookquestions/1147_im19.JPG BM = MC             ... (1)

     And AM = MD      ... (2)  
     Subtracting equations (2) from (1), we have  
     AM - BM = MD - MC  
http://images.topperlearning.com/topper/bookquestions/1147_13fb2c7abb7fb90f3d38dc838d646ea3.png AB = CD

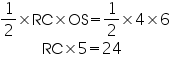
Solution 5

Draw perpendiculars OA and OB on RS and SM respectively.  
Let R, S and M be the position of Reshma, Salma and Mandip respectively.



   AR = AS = http://images.topperlearning.com/topper/bookquestions/1148_e7138cd17cc5cbab052d07b344a6561a.png = 3cm

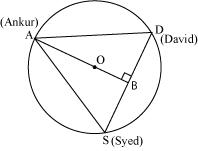
   OR = OS = OM = 5 m     (radii of circle)  
    In OAR  
    OA2 + AR2 = OR2              
    OA2 + (3 m)2 = (5 m)2  
    OA2 = (25 - 9) m2 = 16 m2  
    OA = 4 m                                 
    We know that in an isosceles triangle altitude divides the base, so in http://images.topperlearning.com/topper/bookquestions/1148_ec7fd5b2b4109644a398eba4d25ebf00.pngRSM  
    http://images.topperlearning.com/topper/bookquestions/1148_4934c411dbc1a55c0b7875b3e0fc862a.pngRCS will be of 90o and RC = CM               
    Area of http://images.topperlearning.com/topper/bookquestions/1148_ec7fd5b2b4109644a398eba4d25ebf00.pngORS =   http://images.topperlearning.com/topper/bookquestions/1148_fcdbbc6bbc2ca084b11561f01b4ff909.pnghttp://images.topperlearning.com/topper/bookquestions/1148_b9ccb383b0c73c26cfb3ff194a86e235.pngOAhttp://images.topperlearning.com/topper/bookquestions/1148_b9ccb383b0c73c26cfb3ff194a86e235.pngRS

  
                      RC = 4.8

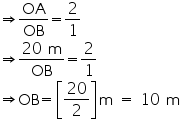
    RM = 2RC = 2(4.8)= 9.6

    So, distance between Reshma and Mandip is 9.6 m.

Solution 6

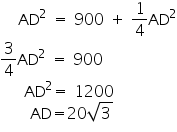


    Given that AS = SD = DA  
             So, ASD is a equilateral triangle  
             OA (radius) = 20 m.  
Medians of equilateral triangle pass through the circum centre (O) of the equilateral triangle ABC.  
We also know that median intersect each other at the 2: 1. As AB is the median of equilateral triangle ABC, we can write



http://images.topperlearning.com/topper/bookquestions/1149_im19.JPG AB = OA + OB = (20 + 10) m = 30 m.

    In http://images.topperlearning.com/topper/bookquestions/1149_ec7fd5b2b4109644a398eba4d25ebf00.pngABD  
  
    AD2 = AB2 + BD2  
    AD2 = (30)2 +   http://images.topperlearning.com/topper/bookquestions/1149_05a5194335b1c58ef14cc1f0b10e0df9.png

         
    So, length of string of each phone will be http://images.topperlearning.com/topper/bookquestions/1149_c5c7af935a78ade28040a27273ff50e4.png m.

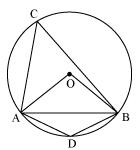
Chapter 10 - Circles Excercise Ex. 10.5

Solution 1

We may observe that  
    http://images.topperlearning.com/topper/bookquestions/1151_4934c411dbc1a55c0b7875b3e0fc862a.pngAOC = http://images.topperlearning.com/topper/bookquestions/1151_4934c411dbc1a55c0b7875b3e0fc862a.pngAOB + http://images.topperlearning.com/topper/bookquestions/1151_4934c411dbc1a55c0b7875b3e0fc862a.pngBOC  
        = 60o + 30o  
        = 90o  
We know that angle subtended by an arc at centre is double the angle subtended by it any point on the remaining part of the circle.

http://images.topperlearning.com/topper/bookquestions/1151_21b105174c84aaff394604cd4649cc6b.png

Solution 2



In http://images.topperlearning.com/topper/bookquestions/1153_ec7fd5b2b4109644a398eba4d25ebf00.pngOAB  
    AB = OA = OB = radius  
http://images.topperlearning.com/topper/bookquestions/1153_im19.JPGhttp://images.topperlearning.com/topper/bookquestions/1153_ec7fd5b2b4109644a398eba4d25ebf00.pngOAB is an equilateral triangle.

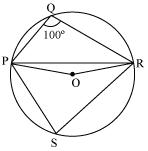
So, each interior angle of this triangle will be of 60o

http://images.topperlearning.com/topper/bookquestions/1153_im19.JPGhttp://images.topperlearning.com/topper/bookquestions/1153_4934c411dbc1a55c0b7875b3e0fc862a.pngAOB = 60o

Now,  http://images.topperlearning.com/topper/bookquestions/1153_bd4e9e438b3f4f6a0cdd95b6666f734c.png

In cyclic quadrilateral ACBD  
http://images.topperlearning.com/topper/bookquestions/1153_4934c411dbc1a55c0b7875b3e0fc862a.pngACB + http://images.topperlearning.com/topper/bookquestions/1153_4934c411dbc1a55c0b7875b3e0fc862a.pngADB = 180o        (Opposite angle in cyclic quadrilateral)  
http://images.topperlearning.com/topper/bookquestions/1153_2457d20c0787e1fb421665bd02d123a1.pngADB = 180o - 30o = 150o  
So, angle subtended by this chord at a point on major arc and minor arc are 30o and 150o respectively.

Solution 3



Consider PR as a chord of circle.  
Take any point S on major arc of circle.  
Now PQRS is a cyclic quadrilateral.

http://images.topperlearning.com/topper/bookquestions/1155_4934c411dbc1a55c0b7875b3e0fc862a.pngPQR + http://images.topperlearning.com/topper/bookquestions/1155_4934c411dbc1a55c0b7875b3e0fc862a.pngPSR = 180o                    (Opposite angles of cyclic quadrilateral)  
http://images.topperlearning.com/topper/bookquestions/1155_86e9f4603b1f61504655f58bc01b32c4.pngPSR = 180o - 100o = 80o  
We know that angle subtended by an arc at centre is double the angle subtended by it any point on the remaining part of the circle.  
http://images.topperlearning.com/topper/bookquestions/1155_im19.JPG http://images.topperlearning.com/topper/bookquestions/1155_4934c411dbc1a55c0b7875b3e0fc862a.pngPOR = 2http://images.topperlearning.com/topper/bookquestions/1155_4934c411dbc1a55c0b7875b3e0fc862a.pngPSR = 2 (80o) = 160o

In http://images.topperlearning.com/topper/bookquestions/1155_ec7fd5b2b4109644a398eba4d25ebf00.pngPOR  
OP = OR                                        (radii of same circle)

http://images.topperlearning.com/topper/bookquestions/1155_im19.JPG http://images.topperlearning.com/topper/bookquestions/1155_4934c411dbc1a55c0b7875b3e0fc862a.pngOPR = http://images.topperlearning.com/topper/bookquestions/1155_4934c411dbc1a55c0b7875b3e0fc862a.pngORP                         (Angles opposite equal sides of a triangle)

http://images.topperlearning.com/topper/bookquestions/1155_4934c411dbc1a55c0b7875b3e0fc862a.pngOPR + http://images.topperlearning.com/topper/bookquestions/1155_4934c411dbc1a55c0b7875b3e0fc862a.pngORP + http://images.topperlearning.com/topper/bookquestions/1155_4934c411dbc1a55c0b7875b3e0fc862a.pngPOR = 180o    (Angle sum property of a triangle)

2 http://images.topperlearning.com/topper/bookquestions/1155_4934c411dbc1a55c0b7875b3e0fc862a.pngOPR + 160o= 180o  
2 http://images.topperlearning.com/topper/bookquestions/1155_4934c411dbc1a55c0b7875b3e0fc862a.pngOPR = 180o - 160o = 20o

http://images.topperlearning.com/topper/bookquestions/1155_4934c411dbc1a55c0b7875b3e0fc862a.pngOPR = 10o

Solution 4

In http://images.topperlearning.com/topper/bookquestions/1156_ec7fd5b2b4109644a398eba4d25ebf00.pngABC

http://images.topperlearning.com/topper/bookquestions/1156_4934c411dbc1a55c0b7875b3e0fc862a.pngBAC + http://images.topperlearning.com/topper/bookquestions/1156_4934c411dbc1a55c0b7875b3e0fc862a.pngABC + http://images.topperlearning.com/topper/bookquestions/1156_4934c411dbc1a55c0b7875b3e0fc862a.pngACB = 180o     (Angle sum property of a triangle)  
http://images.topperlearning.com/topper/bookquestions/1156_2457d20c0787e1fb421665bd02d123a1.pngBAC + 69o + 31o = 180o

http://images.topperlearning.com/topper/bookquestions/1156_2457d20c0787e1fb421665bd02d123a1.pngBAC = 180o - 100º  
http://images.topperlearning.com/topper/bookquestions/1156_2457d20c0787e1fb421665bd02d123a1.pngBAC = 80o

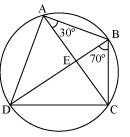
http://images.topperlearning.com/topper/bookquestions/1156_4934c411dbc1a55c0b7875b3e0fc862a.pngBDC = http://images.topperlearning.com/topper/bookquestions/1156_4934c411dbc1a55c0b7875b3e0fc862a.pngBAC = 80o                    (Angles in same segment of circle are equal)

Solution 5

In http://images.topperlearning.com/topper/bookquestions/1158_ec7fd5b2b4109644a398eba4d25ebf00.pngCDE  
http://images.topperlearning.com/topper/bookquestions/1158_4934c411dbc1a55c0b7875b3e0fc862a.pngCDE + http://images.topperlearning.com/topper/bookquestions/1158_4934c411dbc1a55c0b7875b3e0fc862a.pngDCE = http://images.topperlearning.com/topper/bookquestions/1158_4934c411dbc1a55c0b7875b3e0fc862a.pngCEB        (Exterior angle)  
http://images.topperlearning.com/topper/bookquestions/1158_2457d20c0787e1fb421665bd02d123a1.pngCDE + 20o = 130o

http://images.topperlearning.com/topper/bookquestions/1158_2457d20c0787e1fb421665bd02d123a1.pngCDE = 110o  
But http://images.topperlearning.com/topper/bookquestions/1158_4934c411dbc1a55c0b7875b3e0fc862a.pngBAC = http://images.topperlearning.com/topper/bookquestions/1158_4934c411dbc1a55c0b7875b3e0fc862a.pngCDE               (Angles in same segment of circle)  
http://images.topperlearning.com/topper/bookquestions/1158_2457d20c0787e1fb421665bd02d123a1.pngBAC = 110o

Solution 6



For chord CD

http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngCBD = http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngCAD                    (Angles in same segment)

http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngCAD = 70o

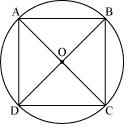
http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngBAD = http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngBAC + http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngCAD = 30o + 70o = 100o  
http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngBCD + http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngBAD = 180o        (Opposite angles of a cyclic quadrilateral)  
http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngBCD + 100o = 180o  
http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngBCD = 80o  
In http://images.topperlearning.com/topper/bookquestions/1159_ec7fd5b2b4109644a398eba4d25ebf00.pngABC

AB = BC                               (given)  
 http://images.topperlearning.com/topper/bookquestions/1159_im19.JPGhttp://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngBCA = http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngCAB               (Angles opposite to equal sides of a triangle)  
http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngBCA = 30o  
We have http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngBCD = 80o  
http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngBCA + http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngACD = 80o  
30o + http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngACD = 80o

http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngACD = 50o

http://images.topperlearning.com/topper/bookquestions/1159_4934c411dbc1a55c0b7875b3e0fc862a.pngECD = 50o

Solution 7



Let ABCD a cyclic quadrilateral having diagonals as BD and AC intersecting each other at point O.

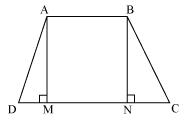
http://images.topperlearning.com/topper/bookquestions/1160_3722fd1f766e804fad6fc84694ed27ad.png    (Consider BD as a chord)

http://images.topperlearning.com/topper/bookquestions/1160_4934c411dbc1a55c0b7875b3e0fc862a.pngBCD + http://images.topperlearning.com/topper/bookquestions/1160_4934c411dbc1a55c0b7875b3e0fc862a.pngBAD = 180o            (Cyclic quadrilateral)  
    http://images.topperlearning.com/topper/bookquestions/1160_4934c411dbc1a55c0b7875b3e0fc862a.pngBCD = 180o - 90o = 90o

http://images.topperlearning.com/topper/bookquestions/1160_5ef77821ce67cada25c8018afa0461f7.png       (Considering AC as a chord)

http://images.topperlearning.com/topper/bookquestions/1160_4934c411dbc1a55c0b7875b3e0fc862a.pngADC + http://images.topperlearning.com/topper/bookquestions/1160_4934c411dbc1a55c0b7875b3e0fc862a.pngABC = 180o            (Cyclic quadrilateral)  
    90o + http://images.topperlearning.com/topper/bookquestions/1160_4934c411dbc1a55c0b7875b3e0fc862a.pngABC = 180o  
    http://images.topperlearning.com/topper/bookquestions/1160_4934c411dbc1a55c0b7875b3e0fc862a.pngABC = 90o  
    Here, each interior angle of cyclic quadrilateral is of 90o. Hence it is a rectangle.

Solution 8



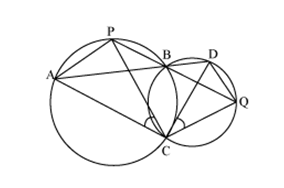
Consider a trapezium ABCD with AB | |CD and BC = AD Draw AM http://images.topperlearning.com/topper/bookquestions/1164_cd0ab85520b5ae722383bdcc2bb17bd8.png CD and BN http://images.topperlearning.com/topper/bookquestions/1164_cd0ab85520b5ae722383bdcc2bb17bd8.png CD  
In http://images.topperlearning.com/topper/bookquestions/1164_ec7fd5b2b4109644a398eba4d25ebf00.pngAMD and http://images.topperlearning.com/topper/bookquestions/1164_ec7fd5b2b4109644a398eba4d25ebf00.pngBNC  
 AD = BC                                 (Given)  
http://images.topperlearning.com/topper/bookquestions/1164_4934c411dbc1a55c0b7875b3e0fc862a.pngAMD = http://images.topperlearning.com/topper/bookquestions/1164_4934c411dbc1a55c0b7875b3e0fc862a.pngBNC                      (By construction each is 90o)  
AM = BM    (Perpendicular distance between two parallel lines is same)  
http://images.topperlearning.com/topper/bookquestions/1164_im19.JPGhttp://images.topperlearning.com/topper/bookquestions/1164_ec7fd5b2b4109644a398eba4d25ebf00.pngAMD http://images.topperlearning.com/topper/bookquestions/1164_6cf26576c060f5f190df510e5535ff3c.png BNC              (RHS congruence rule)

http://images.topperlearning.com/topper/bookquestions/1164_im19.JPGhttp://images.topperlearning.com/topper/bookquestions/1164_4934c411dbc1a55c0b7875b3e0fc862a.pngADC = http://images.topperlearning.com/topper/bookquestions/1164_4934c411dbc1a55c0b7875b3e0fc862a.pngBCD                   (CPCT)    ... (1)

http://images.topperlearning.com/topper/bookquestions/1164_4934c411dbc1a55c0b7875b3e0fc862a.pngBAD and http://images.topperlearning.com/topper/bookquestions/1164_4934c411dbc1a55c0b7875b3e0fc862a.pngADC are on same side of transversal AD

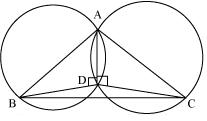
http://images.topperlearning.com/topper/bookquestions/1164_4934c411dbc1a55c0b7875b3e0fc862a.pngBAD + http://images.topperlearning.com/topper/bookquestions/1164_4934c411dbc1a55c0b7875b3e0fc862a.pngADC = 180o                ... (2)      
http://images.topperlearning.com/topper/bookquestions/1164_4934c411dbc1a55c0b7875b3e0fc862a.pngBAD + http://images.topperlearning.com/topper/bookquestions/1164_4934c411dbc1a55c0b7875b3e0fc862a.pngBCD = 180o          [Using equation (1)]  
This equation shows that the opposite angles are supplementary.  
So, ABCD is a cyclic quadrilateral.

Solution 9

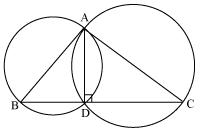


    Join chords AP and DQ  
    For chord AP  
    http://images.topperlearning.com/topper/bookquestions/1165_4934c411dbc1a55c0b7875b3e0fc862a.pngPBA = http://images.topperlearning.com/topper/bookquestions/1165_4934c411dbc1a55c0b7875b3e0fc862a.pngACP         (Angles in same segment)        ... (1)    
    For chord DQ  
    http://images.topperlearning.com/topper/bookquestions/1165_4934c411dbc1a55c0b7875b3e0fc862a.pngDBQ = http://images.topperlearning.com/topper/bookquestions/1165_4934c411dbc1a55c0b7875b3e0fc862a.pngQCD         (Angles in same segment)        ... (2)     
    ABD and PBQ are line segments intersecting at B.  
    http://images.topperlearning.com/topper/bookquestions/1165_im19.JPGhttp://images.topperlearning.com/topper/bookquestions/1165_4934c411dbc1a55c0b7875b3e0fc862a.pngPBA = http://images.topperlearning.com/topper/bookquestions/1165_4934c411dbc1a55c0b7875b3e0fc862a.pngDBQ         (Vertically opposite angles)        ... (3)      
    From equations (1), (2) and (3), we have  
    http://images.topperlearning.com/topper/bookquestions/1165_4934c411dbc1a55c0b7875b3e0fc862a.pngACP = http://images.topperlearning.com/topper/bookquestions/1165_4934c411dbc1a55c0b7875b3e0fc862a.pngQCD

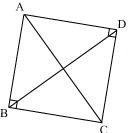
Solution 10



Consider a http://images.topperlearning.com/topper/bookquestions/1166_ec7fd5b2b4109644a398eba4d25ebf00.pngABC  
Two circles are drawn while taking AB and AC as diameter.  
 Let they intersect each other at D and let D does not lie on BC.  
 Join AD  
    http://images.topperlearning.com/topper/bookquestions/1166_4934c411dbc1a55c0b7875b3e0fc862a.pngADB = 90o            (Angle subtend by semicircle)  
    http://images.topperlearning.com/topper/bookquestions/1166_4934c411dbc1a55c0b7875b3e0fc862a.pngADC = 90o            (Angle subtend by semicircle)  
    http://images.topperlearning.com/topper/bookquestions/1166_4934c411dbc1a55c0b7875b3e0fc862a.pngBDC = http://images.topperlearning.com/topper/bookquestions/1166_4934c411dbc1a55c0b7875b3e0fc862a.pngADB + http://images.topperlearning.com/topper/bookquestions/1166_4934c411dbc1a55c0b7875b3e0fc862a.pngADC = 90o + 90o = 180o  
 Hence BDC is straight line and our assumption was wrong.  
 Thus, Point D lies on third side BC of http://images.topperlearning.com/topper/bookquestions/1166_ec7fd5b2b4109644a398eba4d25ebf00.pngABC



Solution 11



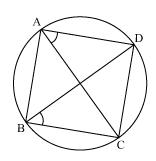
In http://images.topperlearning.com/topper/bookquestions/1168_ec7fd5b2b4109644a398eba4d25ebf00.pngABC

http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngABC + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngBCA + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngCAB = 180o    (Angle sum property of a triangle)  
http://images.topperlearning.com/topper/bookquestions/1168_13fb2c7abb7fb90f3d38dc838d646ea3.png 90o + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngBCA + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngCAB = 180o  
http://images.topperlearning.com/topper/bookquestions/1168_13fb2c7abb7fb90f3d38dc838d646ea3.png http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngBCA + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngCAB = 90o        ... (1)  
In http://images.topperlearning.com/topper/bookquestions/1168_ec7fd5b2b4109644a398eba4d25ebf00.pngADC

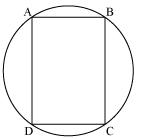
http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngCDA + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngACD + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngDAC = 180o    (Angle sum property of a triangle)  
http://images.topperlearning.com/topper/bookquestions/1168_13fb2c7abb7fb90f3d38dc838d646ea3.png 90o + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngACD + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngDAC = 180o  
http://images.topperlearning.com/topper/bookquestions/1168_13fb2c7abb7fb90f3d38dc838d646ea3.png http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngACD + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngDAC = 90o        ... (2)  
Adding equations (1) and (2), we have

http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngBCA + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngCAB + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngACD + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngDAC = 180o

http://images.topperlearning.com/topper/bookquestions/1168_13fb2c7abb7fb90f3d38dc838d646ea3.png (http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngBCA + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngACD) + (http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngCAB + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngDAC) = 180o http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngBCD + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngDAB = 180o        ... (3)  
    But it is given that  
http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngB + http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngD = 90o + 90o = 180o        ... (4)  
From equations (3) and (4), we can see that quadrilateral ABCD is having sum of measures of opposite angles as 180o.  
So, it is a cyclic quadrilateral.  
Consider chord CD.  
Now, http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngCAD = http://images.topperlearning.com/topper/bookquestions/1168_4934c411dbc1a55c0b7875b3e0fc862a.pngCBD                      (Angles in same segment)



Solution 12



Let ABCD be a cyclic parallelogram.  
    http://images.topperlearning.com/topper/bookquestions/1169_4934c411dbc1a55c0b7875b3e0fc862a.pngA + http://images.topperlearning.com/topper/bookquestions/1169_4934c411dbc1a55c0b7875b3e0fc862a.pngC = 180o     (Opposite angle of cyclic quadrilateral)    ... (1)  
    We know that opposite angles of a parallelogram are equal  
    http://images.topperlearning.com/topper/bookquestions/1169_im19.JPGhttp://images.topperlearning.com/topper/bookquestions/1169_4934c411dbc1a55c0b7875b3e0fc862a.pngA = http://images.topperlearning.com/topper/bookquestions/1169_4934c411dbc1a55c0b7875b3e0fc862a.pngC and http://images.topperlearning.com/topper/bookquestions/1169_4934c411dbc1a55c0b7875b3e0fc862a.pngB = http://images.topperlearning.com/topper/bookquestions/1169_4934c411dbc1a55c0b7875b3e0fc862a.pngD  
    From equation (1)  
    http://images.topperlearning.com/topper/bookquestions/1169_4934c411dbc1a55c0b7875b3e0fc862a.pngA + http://images.topperlearning.com/topper/bookquestions/1169_4934c411dbc1a55c0b7875b3e0fc862a.pngC = 180o  
http://images.topperlearning.com/topper/bookquestions/1169_86e9f4603b1f61504655f58bc01b32c4.pngA + http://images.topperlearning.com/topper/bookquestions/1169_4934c411dbc1a55c0b7875b3e0fc862a.pngA = 180o  
http://images.topperlearning.com/topper/bookquestions/1169_13fb2c7abb7fb90f3d38dc838d646ea3.png2 http://images.topperlearning.com/topper/bookquestions/1169_4934c411dbc1a55c0b7875b3e0fc862a.pngA = 180o  
http://images.topperlearning.com/topper/bookquestions/1169_86e9f4603b1f61504655f58bc01b32c4.pngA = 90o  
Parallelogram ABCD is having its one of interior angles as 90o, so, it is a rectangle.

**PROMOTED CONTENT**

[**This Made Coimbatore Man's Income Soar To Rs. 11,600,000!**](https://storiespace/This_Made_Coimbatore_Mans_Income_Soar_To_Rs._11600000)

[Storiespace](https://storiespace/This_Made_Coimbatore_Mans_Income_Soar_To_Rs._11600000)

[**A Girl From Coimbatore Became a Billionaire Using This Method**](https://storiespace/A_Girl_From_Coimbatore_Became_a_Billionaire_Using_This_Method)

[Storiespace](https://storiespace/A_Girl_From_Coimbatore_Became_a_Billionaire_Using_This_Method)

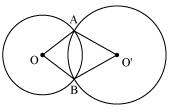
[**A Girl From Coimbatore Became A Billionaire Using This Method**](https://storiespace/A_Girl_From_Coimbatore_Became_A_Billionaire_Using_This_Method)

[Storiespace](https://storiespace/A_Girl_From_Coimbatore_Became_A_Billionaire_Using_This_Method)

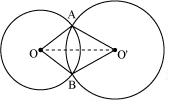
[**वह बहुत गरीब थी लेकिन अब करोडो कमाती है**](https://xn--________-89p4idbr1m9amh0dxddh3c5al6b5dxfcx1mtazdri1u2b8fb/)

Chapter 10 - Circles Excercise Ex. 10.6

Solution 1



Let two circles having their centres as O and intersect each other at point A and B respectively.  
Construction: Let us join OO',

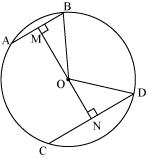


Now in http://images.topperlearning.com/topper/bookquestions/1173_ec7fd5b2b4109644a398eba4d25ebf00.pngAOO'  and http://images.topperlearning.com/topper/bookquestions/1173_ec7fd5b2b4109644a398eba4d25ebf00.pngBOO'  
OA = OB                           (radius of circle 1)  
O'A =  O'B                        (radius of circle 2)  
OO'  = OO'                        (common)

http://images.topperlearning.com/topper/bookquestions/1173_ec7fd5b2b4109644a398eba4d25ebf00.png AOO'  http://images.topperlearning.com/topper/bookquestions/1173_5104e88b0db655a5872db484bf6ee948.pngBOO'         (by SSS congruence rule)  
http://images.topperlearning.com/topper/bookquestions/1173_4934c411dbc1a55c0b7875b3e0fc862a.pngOAO'  = http://images.topperlearning.com/topper/bookquestions/1173_4934c411dbc1a55c0b7875b3e0fc862a.pngOBO'             (by CPCT)  
So, line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Solution 2

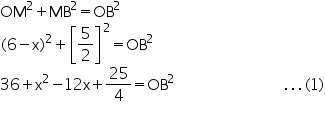
Draw OM http://images.topperlearning.com/topper/bookquestions/1174_cd0ab85520b5ae722383bdcc2bb17bd8.png AB and ON http://images.topperlearning.com/topper/bookquestions/1174_cd0ab85520b5ae722383bdcc2bb17bd8.png CD. Join OB and OD



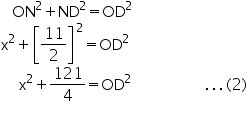
                     (Perpendicular from centre bisects the chord)

http://images.topperlearning.com/topper/bookquestions/1174_c289cae4cf9e108e140ec5f6b5737d60.png

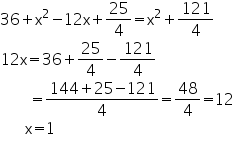
Let ON be x, so OM will be 6 - x  
In http://images.topperlearning.com/topper/bookquestions/1174_ec7fd5b2b4109644a398eba4d25ebf00.pngMOB



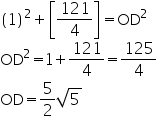
In http://images.topperlearning.com/topper/bookquestions/1174_ec7fd5b2b4109644a398eba4d25ebf00.pngNOD



We have OB = OD             (radii of same circle)  
So, from equation (1) and (2)

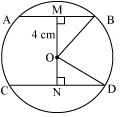


From equation (2)



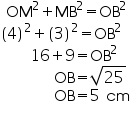
So, radius of circle is found to be http://images.topperlearning.com/topper/bookquestions/1174_9ef840c632c5e9cbe993817bd462b1ff.png cm.

Solution 3



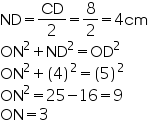
Distance of smaller chord AB from centre of circle = 4 cm.  
OM = 4 cm  
  
http://images.topperlearning.com/topper/bookquestions/1176_cb32bdd39df2ae30ee3a2091478ccd8c.png

In http://images.topperlearning.com/topper/bookquestions/1176_ec7fd5b2b4109644a398eba4d25ebf00.pngOMB



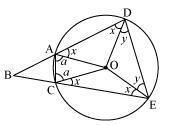
In http://images.topperlearning.com/topper/bookquestions/1176_ec7fd5b2b4109644a398eba4d25ebf00.pngOND

OD=OB=5cm             (radii of same circle)



So, distance of bigger chord from centre is 3 cm.

Solution 4



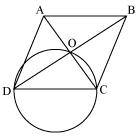
In https://images.topperlearning.com/topper/bookquestions/1177_ec7fd5b2b4109644a398eba4d25ebf00.pngAOD and https://images.topperlearning.com/topper/bookquestions/1177_ec7fd5b2b4109644a398eba4d25ebf00.pngCOE  
    OA = OC             (radii of same circle)  
    OD = OE             (radii of same circle)  
    AD = CE            (given)  
https://images.topperlearning.com/topper/bookquestions/1177_im19.JPG https://images.topperlearning.com/topper/bookquestions/1177_ec7fd5b2b4109644a398eba4d25ebf00.pngAOD https://images.topperlearning.com/topper/bookquestions/1177_5104e88b0db655a5872db484bf6ee948.pngCOE         (SSS congruence rule)

https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngOAD = https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngOCE         (by CPCT)        ... (1)  
https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngODA = https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngOEC         (by CPCT)        ... (2)  
We also have  
https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngOAD = https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngODA        (As OA = OD)        ... (3)  
From equations (1), (2) and (3), we have  
https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngOAD = https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngOCE = https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngODA = https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngOEC  
Let https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngOAD = https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngOCE = https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngODA = https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngOEC = x  
In https://images.topperlearning.com/topper/bookquestions/1177_ec7fd5b2b4109644a398eba4d25ebf00.png OAC,  
OA = OC  
https://images.topperlearning.com/topper/bookquestions/1177_im19.JPG https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngOCA = https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngOAC         (let a)

In https://images.topperlearning.com/topper/bookquestions/1177_ec7fd5b2b4109644a398eba4d25ebf00.png ODE,  
OD = OE  
https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngOED = https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngODE         (let y)  
ADEC is a cyclic quadrilateral  
https://images.topperlearning.com/topper/bookquestions/1177_im19.JPG https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngCAD + https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngDEC = 180o         (opposite angles are supplementary)

x + a + x + y = 180o  
2x + a + y = 180o  
y = 180 - 2x - a                    ... (4)  
But https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngDOE = 180 - 2y  
And https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngAOC = 180 - 2a  
Now, https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngDOE - https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngAOC = 2a - 2y = 2a - 2 (180 - 2x - a)  
             = 4a + 4x - 360o        ... (5)  
Now, https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngBAC + https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngCAD = 180    (Linear pair)  
https://images.topperlearning.com/topper/bookquestions/1177_86e9f4603b1f61504655f58bc01b32c4.pngBAC = 180 - https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngCAD = 180 - (a + x)  
Similarly, https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngACB = 180 - (a + x)  
Now, in https://images.topperlearning.com/topper/bookquestions/1177_ec7fd5b2b4109644a398eba4d25ebf00.pngABC  
https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngABC + https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngBAC + https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngACB = 180    (Angle sum property of a triangle)  
https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngABC = 180 - https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngBAC - https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngACB  
= 180 - (180 - a - x) - (180 - a -x)  
= 2a + 2x - 180  
= https://images.topperlearning.com/topper/bookquestions/1177_fcdbbc6bbc2ca084b11561f01b4ff909.png  [4a + 4x - 360o]  
https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngABC = https://images.topperlearning.com/topper/bookquestions/1177_fcdbbc6bbc2ca084b11561f01b4ff909.png [https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngDOE -  https://images.topperlearning.com/topper/bookquestions/1177_4934c411dbc1a55c0b7875b3e0fc862a.pngAOC]    [Using equation (5)]

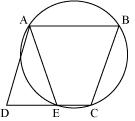
Solution 5



Let ABCD be a rhombus in which diagonals are intersecting at point O and a circle is drawn taking side CD as its diameter.  
We know that angle in a semicircle is of 90o.  
http://images.topperlearning.com/topper/bookquestions/1179_im19.JPG http://images.topperlearning.com/topper/bookquestions/1179_4934c411dbc1a55c0b7875b3e0fc862a.pngCOD = 90o

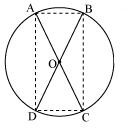
Also in rhombus the diagonals intersect each other at 90o  
http://images.topperlearning.com/topper/bookquestions/1179_4934c411dbc1a55c0b7875b3e0fc862a.pngAOB = http://images.topperlearning.com/topper/bookquestions/1179_4934c411dbc1a55c0b7875b3e0fc862a.pngBOC = http://images.topperlearning.com/topper/bookquestions/1179_4934c411dbc1a55c0b7875b3e0fc862a.pngCOD = http://images.topperlearning.com/topper/bookquestions/1179_4934c411dbc1a55c0b7875b3e0fc862a.pngDOA = 90o  
So, point O has to lie on the circle.

Solution 6



We see that ABCE is a cyclic quadrilateral and in a cyclic quadrilateral sum of opposite angles is 180o  
    http://images.topperlearning.com/topper/bookquestions/1180_4934c411dbc1a55c0b7875b3e0fc862a.pngAEC + http://images.topperlearning.com/topper/bookquestions/1180_4934c411dbc1a55c0b7875b3e0fc862a.pngCBA = 180o  
    http://images.topperlearning.com/topper/bookquestions/1180_4934c411dbc1a55c0b7875b3e0fc862a.pngAEC + http://images.topperlearning.com/topper/bookquestions/1180_4934c411dbc1a55c0b7875b3e0fc862a.pngAED = 180o        (linear pair)  
    http://images.topperlearning.com/topper/bookquestions/1180_4934c411dbc1a55c0b7875b3e0fc862a.pngAED = http://images.topperlearning.com/topper/bookquestions/1180_4934c411dbc1a55c0b7875b3e0fc862a.pngCBA            ... (1)  
    For a parallelogram opposite angles are equal.  
    http://images.topperlearning.com/topper/bookquestions/1180_4934c411dbc1a55c0b7875b3e0fc862a.pngADE = http://images.topperlearning.com/topper/bookquestions/1180_4934c411dbc1a55c0b7875b3e0fc862a.pngCBA            ... (2)  
    From (1) and (2)  
   http://images.topperlearning.com/topper/bookquestions/1180_4934c411dbc1a55c0b7875b3e0fc862a.pngAED = http://images.topperlearning.com/topper/bookquestions/1180_4934c411dbc1a55c0b7875b3e0fc862a.pngADE  
    AD = AE            (angles opposite to equal sides of a triangle)

Solution 7

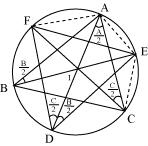


Let two chords AB and CD are intersecting each other at point O.  
In http://images.topperlearning.com/topper/bookquestions/1181_ec7fd5b2b4109644a398eba4d25ebf00.pngAOB and http://images.topperlearning.com/topper/bookquestions/1181_ec7fd5b2b4109644a398eba4d25ebf00.pngCOD  
OA = OC                         (given)  
OB = OD                         (given)  
http://images.topperlearning.com/topper/bookquestions/1181_4934c411dbc1a55c0b7875b3e0fc862a.pngAOB = http://images.topperlearning.com/topper/bookquestions/1181_4934c411dbc1a55c0b7875b3e0fc862a.pngCOD             (vertically opposite angles)  
http://images.topperlearning.com/topper/bookquestions/1181_ec7fd5b2b4109644a398eba4d25ebf00.pngAOB http://images.topperlearning.com/topper/bookquestions/1181_5104e88b0db655a5872db484bf6ee948.pngCOD          (SAS congruence rule)  
AB = CD                        (by CPCT)      
Similarly, we can prove http://images.topperlearning.com/topper/bookquestions/1181_ec7fd5b2b4109644a398eba4d25ebf00.pngAOD http://images.topperlearning.com/topper/bookquestions/1181_5104e88b0db655a5872db484bf6ee948.pngCOB  
http://images.topperlearning.com/topper/bookquestions/1181_im19.JPG AD = CB                     (by CPCT)

Since in quadrilateral ACBD opposite sides are equal in length.  
Hence, ACBD is a parallelogram.  
We know that opposite angles of a parallelogram are equal  
http://images.topperlearning.com/topper/bookquestions/1181_im19.JPG http://images.topperlearning.com/topper/bookquestions/1181_4934c411dbc1a55c0b7875b3e0fc862a.pngA = http://images.topperlearning.com/topper/bookquestions/1181_4934c411dbc1a55c0b7875b3e0fc862a.pngC

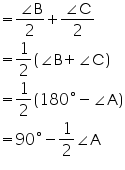
But http://images.topperlearning.com/topper/bookquestions/1181_4934c411dbc1a55c0b7875b3e0fc862a.pngA + http://images.topperlearning.com/topper/bookquestions/1181_4934c411dbc1a55c0b7875b3e0fc862a.pngC = 180o      (ABCD is a cyclic quadrilateral)  
http://images.topperlearning.com/topper/bookquestions/1181_86e9f4603b1f61504655f58bc01b32c4.pngA + http://images.topperlearning.com/topper/bookquestions/1181_4934c411dbc1a55c0b7875b3e0fc862a.pngA = 180o  
 http://images.topperlearning.com/topper/bookquestions/1181_06c5756367ba6f39924ca6c719a6ac6e.png A = 180o  
http://images.topperlearning.com/topper/bookquestions/1181_86e9f4603b1f61504655f58bc01b32c4.pngA = 90o  
As ACBD is a parallelogram and one of its interior angles is 90o, so it is a rectangle.  
http://images.topperlearning.com/topper/bookquestions/1181_4934c411dbc1a55c0b7875b3e0fc862a.pngA is the angle subtended by chord BD. And as http://images.topperlearning.com/topper/bookquestions/1181_4934c411dbc1a55c0b7875b3e0fc862a.pngA = 90o, so BD should be diameter of circle. Similarly AC is diameter of circle.

Solution 8

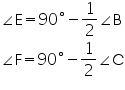


It is given that BE is the bisector of http://images.topperlearning.com/topper/bookquestions/1183_4934c411dbc1a55c0b7875b3e0fc862a.pngB  
http://images.topperlearning.com/topper/bookquestions/1183_im19.JPG http://images.topperlearning.com/topper/bookquestions/1183_4934c411dbc1a55c0b7875b3e0fc862a.pngABE =  http://images.topperlearning.com/topper/bookquestions/1183_4435b3ff64333112f2fb00e244741457.png

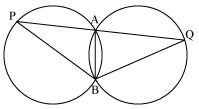
But http://images.topperlearning.com/topper/bookquestions/1183_4934c411dbc1a55c0b7875b3e0fc862a.pngADE = http://images.topperlearning.com/topper/bookquestions/1183_4934c411dbc1a55c0b7875b3e0fc862a.pngABE             (angles in same segment for chord AE)  
http://images.topperlearning.com/topper/bookquestions/1183_86e9f4603b1f61504655f58bc01b32c4.pngADE =  http://images.topperlearning.com/topper/bookquestions/1183_4435b3ff64333112f2fb00e244741457.png  
Similarly, http://images.topperlearning.com/topper/bookquestions/1183_4934c411dbc1a55c0b7875b3e0fc862a.pngACF = http://images.topperlearning.com/topper/bookquestions/1183_4934c411dbc1a55c0b7875b3e0fc862a.pngADF =  http://images.topperlearning.com/topper/bookquestions/1183_5a62f9d474e02fbc2edb5b875612fed3.png    (angle in same segment for chord AF)  
Now, http://images.topperlearning.com/topper/bookquestions/1183_4934c411dbc1a55c0b7875b3e0fc862a.pngD = http://images.topperlearning.com/topper/bookquestions/1183_4934c411dbc1a55c0b7875b3e0fc862a.pngADE + http://images.topperlearning.com/topper/bookquestions/1183_4934c411dbc1a55c0b7875b3e0fc862a.pngADF



Similarly we can prove that



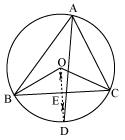
Solution 9



AB is common chord in both congruent circles.  
http://images.topperlearning.com/topper/bookquestions/1184_im19.JPG http://images.topperlearning.com/topper/bookquestions/1184_4934c411dbc1a55c0b7875b3e0fc862a.pngAPB = http://images.topperlearning.com/topper/bookquestions/1184_4934c411dbc1a55c0b7875b3e0fc862a.pngAQB

Now in http://images.topperlearning.com/topper/bookquestions/1184_ec7fd5b2b4109644a398eba4d25ebf00.pngBPQ  
http://images.topperlearning.com/topper/bookquestions/1184_4934c411dbc1a55c0b7875b3e0fc862a.pngAPB = http://images.topperlearning.com/topper/bookquestions/1184_4934c411dbc1a55c0b7875b3e0fc862a.pngAQB    
http://images.topperlearning.com/topper/bookquestions/1184_im19.JPG BP = BQ            (angles opposite to equal sides of a triangle)

Solution 10



Let perpendicular bisector of side BC and angle bisector of http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngA meet at point D.  
 Let perpendicular bisector of side BC intersects it at E.  
  
Perpendicular bisector of side BC will pass through circum centre O of circle. Now, http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngBOC and http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngBAC are the angles subtended by arc BC at the centre and a point A on the remaining part of the circle respectively.  
We also know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.  
http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngBOC = 2 http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngBAC = 2http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngA                 ... (1)  
In http://images.topperlearning.com/topper/bookquestions/1185_ec7fd5b2b4109644a398eba4d25ebf00.pngBOE and http://images.topperlearning.com/topper/bookquestions/1185_ec7fd5b2b4109644a398eba4d25ebf00.pngCOE

OE = OE                                   (common)  
OB = OC                                  (radii of same circle)

http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngOEB = http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngOEC                       (Each 90o as OD http://images.topperlearning.com/topper/bookquestions/1185_cd0ab85520b5ae722383bdcc2bb17bd8.png BC)

http://images.topperlearning.com/topper/bookquestions/1185_im19.JPG http://images.topperlearning.com/topper/bookquestions/1185_ec7fd5b2b4109644a398eba4d25ebf00.pngBOE http://images.topperlearning.com/topper/bookquestions/1185_5104e88b0db655a5872db484bf6ee948.png COE                (RHS congruence rule)

http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngBOE = http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngCOE            (by CPCT)    ... (2)  
But http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngBOE + http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngCOE = http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngBOC

http://images.topperlearning.com/topper/bookquestions/1185_86e9f4603b1f61504655f58bc01b32c4.png BOE +http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngBOE = 2http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.png A        [Using equations (1) and (2)]  
http://images.topperlearning.com/topper/bookquestions/1185_06c5756367ba6f39924ca6c719a6ac6e.pngBOE = 2http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngA

http://images.topperlearning.com/topper/bookquestions/1185_86e9f4603b1f61504655f58bc01b32c4.pngBOE = http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngA

http://images.topperlearning.com/topper/bookquestions/1185_im19.JPGhttp://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngBOE = http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngCOE = http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngA

The perpendicular bisector of side BC and angle bisector of http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngA meet at point D.

http://images.topperlearning.com/topper/bookquestions/1185_im19.JPG http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngBOD = http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngBOE = http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngA                ... (3)

Since AD is the bisector of angle http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngA

http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngBAD =  http://images.topperlearning.com/topper/bookquestions/1185_d26536506333e14e7c2441b63cc33712.png  
http://images.topperlearning.com/topper/bookquestions/1185_13fb2c7abb7fb90f3d38dc838d646ea3.png 2http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngBAD = http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngA                    ... (4)  
From equations (3) and (4), we have  
http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngBOD = 2http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.png BAD  
It is possible only if BD will be a chord of the circle. For this the point D lies on circum circle.    
Therefore, the perpendicular bisector of side BC and angle bisector of http://images.topperlearning.com/topper/bookquestions/1185_4934c411dbc1a55c0b7875b3e0fc862a.pngA meet on the circum circle of triangle ABC.