Chapter 10 - Circles Excercise Ex. 10.1

Solution 1

(i)   The centre of a circle lies in interior of the circle. (exterior/interior)
(ii)  A point, whose distance from the centre of a circle is greater than its radius lies in exterior of

      the circle. (exterior/interior)
(iii) The longest chord of a circle is a diameter of the circle.
(iv) An arc is a semicircle when its ends are the ends of a diameter.
(v) Segment of a circle is the region between an arc and chord of the circle.
(vi) A circle divides the plane, on which it lies, in three parts.

Solution 2

(i)  True, all the points on circle are at equal distance from the centre of circle, and this equal distance

 it called as radius of circle.

(ii) False, on a circle there are infinite points. So, we can draw infinite number of chords of given length. Hence, a circle has infinite number of equal chords.

(iii) False, consider three arcs of same length as AB, BC and CA. Now we may observe that for minor arc BDC. CAB is major arc. So AB, BC and CA are minor arcs of circle.

                                 

(iv) True, let AB be a chord which is twice as long as its radius. In this situation our chord will be passing through centre of circle. So it will be the diameter of circle.

                                  

(v) False, sector is the region between an arc and two radii joining the centre to the end points of the arc as in the given figure OAB is the sector of circle.

                                       

(vi) True, A circle is a two dimensional figure and it can also be referred as plane figure.

Chapter 10 - Circles Excercise Ex. 10.2

Solution 1

A circle is a collection of points which are equidistant from a fix point. This fix point is called as the centre of circle and this equal distance is called as radius of circle. And thus shape of a circle depends on the radius of the circle.

So, if we try to superimpose two circles of equal radius, one each other both circles will cover each other.
So, two circles are congruent if they have equal radius.

Now consider two congruent circles having centre O and O' and two chords AB and CD of equal lengths

                         

Now in AOB and CO'D
AB = CD            (chords of same length)
OA = O'C            (radii of congruent circles)
OB = O'D            (radii of congruent circles)
 AOB  CO'D        (SSS congruence rule)

 AOB = CO'D            (by CPCT)
Hence equal chords of congruent circles subtend equal angles at their centres.

Solution 2

Let us consider two congruent circles (circles of same radius) with centres as O and O'.



In AOB and CO'D
AOB = CO'D        (given)
OA = O'C            (radii of congruent circles)
OB = O'D            (radii of congruent circles)
 AOB CO'D        (SSS congruence rule)
 AB = CD            (by CPCT)
Hence, if chords of congruent circles subtend equal angles at their centres then chords are equal.

Chapter 10 - Circles Excercise Ex. 10.3

Solution 1

Consider the following pair of circles.
(i) circles don't intersect each other at any point, so circles are not having any point in common.

                    

(ii) Circles touch each other only at one point P so there is only 1 point in common.

                     

(iii) Circles touch each other at 1 point X only. So the circles have 1 point in common.

                                   

(iv) These circles intersect each other at two points P and Q. So the circles have two points in common. We may observe that there can be maximum 2 points in common.

                         

We can have a situation in which two congruent circles are superimposed on each other, this situation can be referred as if we are drawing circle two times.

Solution 2

Following are the steps of construction:

Step1. Take the given circle centered at point O.
Step2. Take any two different chords AB and CD of this circle and draw perpendicular bisectors of these

          chords.
Step3. Let these perpendicular bisectors meet at point O. Now, O is the centre of given circle.

                    

Solution 3

Consider two circles centered at point O and O' intersect each other at point A and B respectively.
Join AB. AB is the chord for circle centered at O, so perpendicular bisector of AB will pass through O.
Again AB is also chord of circle centered at O', so, perpendicular bisector of AB will also pass through O'.
Clearly centres of these circles lie on the perpendicular bisector of common chord.

Chapter 10 - Circles Excercise Ex. 10.4

Solution 1

                      

Let radius of circle centered at O and O' be 5 cm and 3 cm respectively.
    OA = OB = 5 cm
    O'A = O'B = 3 cm
    OO' will be the perpendicular bisector of chord AB.
     AC = CB

    Given that OO' = 4 cm
    Let OC be x. so, O'C will be 4 - x
    In OAC
    OA2 = AC2 + OC2
     52 = AC2 + x2
     25 - x2 = AC2            ... (1)
    In O'AC
    O'A2 = AC2 + O'C2
     32 = AC2 + (4 - x)2
     9 = AC2 + 16 + x2 - 8x
     AC2 = - x2 - 7 + 8x        ... (2)
From equations (1) and (2), we have
    25 - x2 = - x2 - 7 + 8x
           8x = 32
             x = 4
So, the common chord will pass through the centre of smaller circle i.e. O'. and hence it will be diameter of smaller circle.

                      

Now, AC2 = 25 - x2 = 25 - 42 = 25 - 16 = 9
 AC = 3 m

The length of the common chord AB = 2 AC = (2  3) m = 6 m

Solution 2

Let PQ and RS are two equal chords of a given circle and there are intersecting each other at point T.



    Draw perpendiculars OV and OU on these chords.
    In OVT and OUT
    OV = OU                                (Equal chords of a circle are equidistant from the centre)
   OVT = OUT                     (Each 90o)
   OT = OT                                (common)

OVT  OUT              (RHS congruence rule)

 VT = UT                             (by CPCT)        ... (1)

    It is given that
    PQ = RS                                           ... ... ... ... (2)



 PV = RU                                    ... ... ...  ... (3)
    On adding equations (1) and (3), we have
    PV + VT = RU + UT
 PT = RT                                    ... ... ...   ... (4)
    On subtracting equation (4) from equation (2), we have
    PQ - PT = RS - RT
 QT = ST                                      ... ... ... ... (5)
    Equations (4) and (5) shows that the corresponding segments of
    chords PQ and RS are congruent to each other.

Solution 3

            

     Let PQ and RS are two equal chords of a given circle and there are intersecting each other at point T.
     Draw perpendiculars OV and OU on these chords.
     In OVT and OUT
     OV = OU                           (Equal chords of a circle are equidistant from the centre)
    OVT = OUT                 (Each 90o)
    OT = OT                            (common)
OVT OUT            (RHS congruence rule)
OTV = OTU                 (by CPCT)

    Hence, the line joining the point of intersection to the centre makes equal angles with the chords.

Solution 4

Let us draw a perpendicular OM on line AD.

                     

    Here, BC is chord of smaller circle and AD is chord of bigger circle.
    We know that the perpendicular drawn from centre of circle bisects the chord.
 BM = MC             ... (1)

     And AM = MD      ... (2)
     Subtracting equations (2) from (1), we have
     AM - BM = MD - MC
 AB = CD

Solution 5

Draw perpendiculars OA and OB on RS and SM respectively.
Let R, S and M be the position of Reshma, Salma and Mandip respectively.

                                    

   AR = AS =  = 3cm

   OR = OS = OM = 5 m     (radii of circle)
    In OAR
    OA2 + AR2 = OR2
    OA2 + (3 m)2 = (5 m)2
    OA2 = (25 - 9) m2 = 16 m2
    OA = 4 m
    We know that in an isosceles triangle altitude divides the base, so in RSM
    RCS will be of 90o and RC = CM
    Area of ORS =   OARS

    
                      RC = 4.8

    RM = 2RC = 2(4.8)= 9.6

    So, distance between Reshma and Mandip is 9.6 m.

Solution 6



    Given that AS = SD = DA
             So, ASD is a equilateral triangle
             OA (radius) = 20 m.
Medians of equilateral triangle pass through the circum centre (O) of the equilateral triangle ABC.
We also know that median intersect each other at the 2: 1. As AB is the median of equilateral triangle ABC, we can write

        

    AB = OA + OB = (20 + 10) m = 30 m.

    In ABD

    AD2 = AB2 + BD2
    AD2 = (30)2 +   

   
    So, length of string of each phone will be  m.

Chapter 10 - Circles Excercise Ex. 10.5

Solution 1

We may observe that
    AOC = AOB + BOC
        = 60o + 30o
        = 90o
We know that angle subtended by an arc at centre is double the angle subtended by it any point on the remaining part of the circle.



Solution 2

                           

In OAB
    AB = OA = OB = radius
OAB is an equilateral triangle.

So, each interior angle of this triangle will be of 60o

AOB = 60o

Now,  

In cyclic quadrilateral ACBD
ACB + ADB = 180o        (Opposite angle in cyclic quadrilateral)
ADB = 180o - 30o = 150o
So, angle subtended by this chord at a point on major arc and minor arc are 30o and 150o respectively.

Solution 3

                               

Consider PR as a chord of circle.
Take any point S on major arc of circle.
Now PQRS is a cyclic quadrilateral.

PQR + PSR = 180o                    (Opposite angles of cyclic quadrilateral)
PSR = 180o - 100o = 80o
We know that angle subtended by an arc at centre is double the angle subtended by it any point on the remaining part of the circle.
 POR = 2PSR = 2 (80o) = 160o

In POR
OP = OR                                        (radii of same circle)

 OPR = ORP                         (Angles opposite equal sides of a triangle)

OPR + ORP + POR = 180o    (Angle sum property of a triangle)

2 OPR + 160o= 180o
2 OPR = 180o - 160o = 20o

OPR = 10o

Solution 4

In ABC

BAC + ABC + ACB = 180o     (Angle sum property of a triangle)
BAC + 69o + 31o = 180o

BAC = 180o - 100º
BAC = 80o

BDC = BAC = 80o                    (Angles in same segment of circle are equal)

Solution 5

In CDE
CDE + DCE = CEB        (Exterior angle)
CDE + 20o = 130o

CDE = 110o
But BAC = CDE               (Angles in same segment of circle)
BAC = 110o

Solution 6

                                

For chord CD

CBD = CAD                    (Angles in same segment)

CAD = 70o

BAD = BAC + CAD = 30o + 70o = 100o
BCD + BAD = 180o        (Opposite angles of a cyclic quadrilateral)
BCD + 100o = 180o
BCD = 80o
In ABC

AB = BC                               (given)
 BCA = CAB               (Angles opposite to equal sides of a triangle)
BCA = 30o
We have BCD = 80o
BCA + ACD = 80o
30o + ACD = 80o

ACD = 50o

ECD = 50o

Solution 7

                          

Let ABCD a cyclic quadrilateral having diagonals as BD and AC intersecting each other at point O.

        (Consider BD as a chord)

    BCD + BAD = 180o            (Cyclic quadrilateral)
    BCD = 180o - 90o = 90o

            (Considering AC as a chord)

   ADC + ABC = 180o            (Cyclic quadrilateral)
    90o + ABC = 180o
    ABC = 90o
    Here, each interior angle of cyclic quadrilateral is of 90o. Hence it is a rectangle.

Solution 8

                            

Consider a trapezium ABCD with AB | |CD and BC = AD Draw AM  CD and BN  CD
In AMD and BNC
 AD = BC                                 (Given)
AMD = BNC                      (By construction each is 90o)
AM = BM    (Perpendicular distance between two parallel lines is same)
AMD  BNC              (RHS congruence rule)

ADC = BCD                   (CPCT)    ... (1)

BAD and ADC are on same side of transversal AD

BAD + ADC = 180o                ... (2)
BAD + BCD = 180o          [Using equation (1)]
This equation shows that the opposite angles are supplementary.
So, ABCD is a cyclic quadrilateral.

Solution 9

                                  

    Join chords AP and DQ
    For chord AP
    PBA = ACP         (Angles in same segment)        ... (1)
    For chord DQ
    DBQ = QCD         (Angles in same segment)        ... (2)
    ABD and PBQ are line segments intersecting at B.
    PBA = DBQ         (Vertically opposite angles)        ... (3)
    From equations (1), (2) and (3), we have
    ACP = QCD

Solution 10

                       

Consider a ABC
Two circles are drawn while taking AB and AC as diameter.
 Let they intersect each other at D and let D does not lie on BC.
 Join AD
    ADB = 90o            (Angle subtend by semicircle)
    ADC = 90o            (Angle subtend by semicircle)
    BDC = ADB + ADC = 90o + 90o = 180o
 Hence BDC is straight line and our assumption was wrong.
 Thus, Point D lies on third side BC of ABC

                              

Solution 11

                     

In ABC

ABC + BCA + CAB = 180o    (Angle sum property of a triangle)
 90o + BCA + CAB = 180o
 BCA + CAB = 90o        ... (1)
In ADC

CDA + ACD + DAC = 180o    (Angle sum property of a triangle)
 90o + ACD + DAC = 180o
 ACD + DAC = 90o        ... (2)
Adding equations (1) and (2), we have

BCA + CAB + ACD + DAC = 180o

 (BCA + ACD) + (CAB + DAC) = 180o BCD + DAB = 180o        ... (3)
    But it is given that
B + D = 90o + 90o = 180o        ... (4)
From equations (3) and (4), we can see that quadrilateral ABCD is having sum of measures of opposite angles as 180o.
So, it is a cyclic quadrilateral.
Consider chord CD.
Now, CAD = CBD                      (Angles in same segment)

                                          

Solution 12

                                       

Let ABCD be a cyclic parallelogram.
    A + C = 180o     (Opposite angle of cyclic quadrilateral)    ... (1)
    We know that opposite angles of a parallelogram are equal
    A = C and B = D
    From equation (1)
    A + C = 180o
A + A = 180o
2 A = 180o
A = 90o
Parallelogram ABCD is having its one of interior angles as 90o, so, it is a rectangle.

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Chapter 10 - Circles Excercise Ex. 10.6

Solution 1

                     

Let two circles having their centres as O and intersect each other at point A and B respectively.
Construction: Let us join OO',

                         

Now in AOO'  and BOO'
OA = OB                           (radius of circle 1)
O'A =  O'B                        (radius of circle 2)
OO'  = OO'                        (common)

 AOO'  BOO'         (by SSS congruence rule)
OAO'  = OBO'             (by CPCT)
So, line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Solution 2

Draw OM  AB and ON  CD. Join OB and OD

                      

                     (Perpendicular from centre bisects the chord)



Let ON be x, so OM will be 6 - x
In MOB



In NOD



We have OB = OD             (radii of same circle)
So, from equation (1) and (2)



From equation (2)



So, radius of circle is found to be  cm.

Solution 3

                                             

Distance of smaller chord AB from centre of circle = 4 cm.
OM = 4 cm



In OMB



In OND

OD=OB=5cm             (radii of same circle)



So, distance of bigger chord from centre is 3 cm.

Solution 4

                                 

In AOD and COE
    OA = OC             (radii of same circle)
    OD = OE             (radii of same circle)
    AD = CE            (given)
 AOD COE         (SSS congruence rule)

OAD = OCE         (by CPCT)        ... (1)
ODA = OEC         (by CPCT)        ... (2)
We also have
OAD = ODA        (As OA = OD)        ... (3)
From equations (1), (2) and (3), we have
OAD = OCE = ODA = OEC
Let OAD = OCE = ODA = OEC = x
In  OAC,
OA = OC
 OCA = OAC         (let a)

In  ODE,
OD = OE
OED = ODE         (let y)
ADEC is a cyclic quadrilateral
 CAD + DEC = 180o         (opposite angles are supplementary)

x + a + x + y = 180o
2x + a + y = 180o
y = 180 - 2x - a                    ... (4)
But DOE = 180 - 2y
And AOC = 180 - 2a
Now, DOE - AOC = 2a - 2y = 2a - 2 (180 - 2x - a)
             = 4a + 4x - 360o        ... (5)
Now, BAC + CAD = 180    (Linear pair)
BAC = 180 - CAD = 180 - (a + x)
Similarly, ACB = 180 - (a + x)
Now, in ABC
ABC + BAC + ACB = 180    (Angle sum property of a triangle)
ABC = 180 - BAC - ACB
= 180 - (180 - a - x) - (180 - a -x)
= 2a + 2x - 180
=   [4a + 4x - 360o]
ABC =  [DOE -  AOC]    [Using equation (5)]

Solution 5

                              

Let ABCD be a rhombus in which diagonals are intersecting at point O and a circle is drawn taking side CD as its diameter.
We know that angle in a semicircle is of 90o.
 COD = 90o

Also in rhombus the diagonals intersect each other at 90o
AOB = BOC = COD = DOA = 90o
So, point O has to lie on the circle.

Solution 6

                               

We see that ABCE is a cyclic quadrilateral and in a cyclic quadrilateral sum of opposite angles is 180o
    AEC + CBA = 180o
    AEC + AED = 180o        (linear pair)
    AED = CBA            ... (1)
    For a parallelogram opposite angles are equal.
    ADE = CBA            ... (2)
    From (1) and (2)
   AED = ADE
    AD = AE            (angles opposite to equal sides of a triangle)

Solution 7

                                      

Let two chords AB and CD are intersecting each other at point O.
In AOB and COD
OA = OC                         (given)
OB = OD                         (given)
AOB = COD             (vertically opposite angles)
AOB COD          (SAS congruence rule)
AB = CD                        (by CPCT)
Similarly, we can prove AOD COB
 AD = CB                     (by CPCT)

Since in quadrilateral ACBD opposite sides are equal in length.
Hence, ACBD is a parallelogram.
We know that opposite angles of a parallelogram are equal
 A = C

But A + C = 180o      (ABCD is a cyclic quadrilateral)
A + A = 180o
  A = 180o
A = 90o
As ACBD is a parallelogram and one of its interior angles is 90o, so it is a rectangle.
A is the angle subtended by chord BD. And as A = 90o, so BD should be diameter of circle. Similarly AC is diameter of circle.

Solution 8

                              

It is given that BE is the bisector of B
 ABE =  

But ADE = ABE             (angles in same segment for chord AE)
ADE =  
Similarly, ACF = ADF =      (angle in same segment for chord AF)
Now, D = ADE + ADF

               

Similarly we can prove that



Solution 9

                                  

AB is common chord in both congruent circles.
 APB = AQB

Now in BPQ
APB = AQB
 BP = BQ            (angles opposite to equal sides of a triangle)

Solution 10

                                        

Let perpendicular bisector of side BC and angle bisector of A meet at point D.
 Let perpendicular bisector of side BC intersects it at E.

Perpendicular bisector of side BC will pass through circum centre O of circle. Now, BOC and BAC are the angles subtended by arc BC at the centre and a point A on the remaining part of the circle respectively.
We also know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
BOC = 2 BAC = 2A                 ... (1)
In BOE and COE

OE = OE                                   (common)
OB = OC                                  (radii of same circle)

OEB = OEC                       (Each 90o as OD  BC)

 BOE  COE                (RHS congruence rule)

BOE = COE            (by CPCT)    ... (2)
But BOE + COE = BOC

 BOE +BOE = 2 A        [Using equations (1) and (2)]
BOE = 2A

BOE = A

 BOE = COE = A

The perpendicular bisector of side BC and angle bisector of A meet at point D.

 BOD = BOE = A                ... (3)

Since AD is the bisector of angle A

BAD =  
 2BAD = A                    ... (4)
From equations (3) and (4), we have
BOD = 2 BAD
It is possible only if BD will be a chord of the circle. For this the point D lies on circum circle.
Therefore, the perpendicular bisector of side BC and angle bisector of A meet on the circum circle of triangle ABC.